Click to prove you're human

Trigonometric identities are equations involving trigonometric functions that hold true for all values of the variables within their domains. There are many such identities, either involving the sides of a right-angled triangle, its angle, or both. They are based on the six fundamental trigonometric functions: sine (sin), cosine (cos), tangent (tan), cosecant (cosec), secant (sec), and cotangent (cot). All the identities are derived from the six trigonometric functions and are used to simplify expressions, verify equations, and solve trigonometric problems. Trigonometric identities are classified based on the type of relationships they describe among trigonometric functions. These identities express the reciprocal relationships between sine, cosine, tangent, and their corresponding co-functions. ${\sin \theta =\dfrac{1}{\text{cosec}\, \theta }}$ ${\text{cosec}\, \theta =\dfrac{1}{\sin \theta }}$ ${\cos \theta =\dfrac{1}{\sec \theta }}$ ${\sec \theta =\dfrac{1}{\cos \theta }}$ ${\tan \theta =\dfrac{1}{\cot \theta }}$ ${\cot \theta =\dfrac{1}{\tan \theta }}$ There are 3 Pythagorean identities in trigonometry, and they are based on the right-angled triangle rule or Pythagorean theorem. sin2 θ + cos2 θ = 1 1 + tan2 θ = sec2 θ 1 + cot2 θ = cosec2 θ Let us consider a right-angled triangle ABC, which is right-angled at C, as shown. Pythagorean Identity Proof 1 By applying the Pythagoras theorem to this triangle, we get Opposite2 + Adjacent2 = Hypotenuse2 .....(i) On dividing both sides of equation (i) by Hypotenuse2, we get ${\dfrac{Opposite^{2}}{Hypotenuse^{2}}+\dfrac{Adjacent^{2}}{Hypotenuse^{2}}=\dfrac{Hypotenuse^{2}}{Hypotenuse^{2}}}$ From the definitions of the trigonometric ratios, ⇒ sin2 θ + cos2 θ = 1 As we know from the right-angled triangle rule, Opposite2 + Adjacent2 = Hypotenuse2 .....(i) Dividing both sides of equation (i) by Adjacent2, we get ${\dfrac{Opposite^{2}}{Adjacent^{2}}+\dfrac{Adjacent^{2}}{Adjacent^{2}}=\dfrac{Hypotenuse^{2}}{Adjacent^{2}}}$ From the definitions of the trigonometric ratios, ⇒ 1 + tan2 θ = sec2 θ Similarly, Dividing both sides of equation (i) by Opposite2, we get ${\dfrac{Opposite^{2}}{Opposite^{2}}+\dfrac{Adjacent^{2}}{Opposite^{2}}=\dfrac{Hypotenuse^{2}}{Opposite^{2}}}$ By using the definitions of the trigonometric ratios, ⇒ 1 + cot2 θ = cosec2 θ These identities define tangent and cotangent in terms of sine and cosine: tan θ = ${\dfrac{\sin \theta }{\cos \theta }}$ cot θ = ${\dfrac{\cos \theta }{\sin \theta }}$ Complementary identities (also known as co-function identities) show how the function of an angle θ relates to the co-function of its complement angle (90° – θ): sin (90° – θ) = cos θ cos (90° – θ) = sin θ tan (90° – θ) = cot θ cot (90° – θ) = tan θ sec (90° – θ) = cosec θ cosec (90° – θ) = sec θ These identities represent how the trigonometric values of an angle θ are related to its supplementary angle (180° – θ): sin (180° – θ) = sinθ cos (180° – θ) = -cos θ tan (180° – θ) = -tan θ cot (180° – θ) = -cot θ sec (180° – θ)= -sec θ cosec (180° – θ) = cosec θ Even-odd identities describe the behavior of trigonometric functions for opposite angles (−θ) and highlight their symmetry properties. These identities classify trigonometric functions as either even or odd based on how their values change with the sign of the angle. sin (-θ) = -sin θ (odd function) cos (-θ) = cos θ (even function) tan (-θ) = – tan θ (odd function) cot (-θ) = -cot θ (odd function) sec (-θ) = sec θ (even function) cosec (-θ) = -cosec θ (odd function) Trigonometric functions are periodic, which means they repeat themselves after a regular interval, called periods. Each function has a different period. sin (2nπ + θ) = sin θ cos (2nπ + θ) = cos θ tan (nπ + θ) = tan θ cot (nπ + θ) = cot θ sec (2nπ + θ) = sec θ cosec (2nπ + θ) = cosec θ Here, n ∈ ℤ (set of all integers) Note: The sine, cosine, and their reciprocals (cosecant and secant) functions have a period of 2π or 360°. Moreover, the tangent and cotangent functions have a period of π or 180°. The identities involving the sum or difference of two angles for the sine, cosine, and tangent functions are as follows: sin (A + B) = sin A cos B + cos A sin B sin (A – B) = sin A cos B – cos A sin B cos (A + B) = cos A cos B – sin A sin B cos (A – B) = cos A cos B + sin A sin B tan (A + B) = ${\dfrac{\tan A+\tan B}{1-\tan A\tan B}}$ tan (A – B) = ${\dfrac{\tan A-\tan B}{1+\tan A\tan B}}$ If the angles of the six trigonometric functions are doubled, then the identities are: sin 2θ = 2sin θ · cos θ = ${\dfrac{2\tan \theta }{1-\tan ^{2}\theta }}$ cos 2θ = cos2 θ – sin2 θ = ${\dfrac{1-\tan ^{2}\theta }{1+\tan ^{2}\theta }}$ cos 2θ = 2cos2 θ – 1 = 1 – 2 sin2 θ tan 2θ = ${\dfrac{2\tan \theta }{1+\tan ^{2}\theta }}$ sec 2θ = ${\dfrac{\sec ^{2}\theta }{2-\sec ^{2}\theta }}$ cosec 2θ = ${\dfrac{\sec \theta \cdot \text{cosec}\, \theta }{2}}$ cot 2θ = ${\dfrac{\cot ^{2}\theta -1}{2\cot \theta }}$ If the angles of the six trigonometric functions are halved, then the identities are: ${\sin \dfrac{\theta }{2}=\pm \sqrt{\dfrac{1-\cos \theta }{2}}}$ ${\cos \dfrac{\theta }{2}=\pm \sqrt{\dfrac{1+\cos \theta }{2}}}$ ${\tan \dfrac{\theta }{2}=\pm \sqrt{\dfrac{1-\cos \theta }{1+\cos \theta }}}$ When the angle of a trigonometric function is three times a given angle (3θ), the corresponding identities are: sin 3θ = 3sin θ – 4sin3 θ cos 3θ = 4cos3 θ – 3cos θ tan 3θ = ${\dfrac{3\tan \theta -\tan ^{3}\theta }{1-3\tan ^{2}\theta }}$ The product-to-sum identities express products of trigonometric functions as their sums or differences: 2sin A · cos B = sin(A + B) + sin(A – B) 2cos A · cos B = cos(A + B) + cos(A – B) 2sin A · sin B = cos(A – B) – cos(A + B) The sum-to-product identities express sums of trigonometric functions as products of those functions: ${\sin A+\sin B=2\sin \left(\dfrac{A+B}{2}\right) \cos \left(\dfrac{A-B}{2}\right)}$ ${\sin A-\sin B=2\sin \left(\dfrac{A-B}{2}\right) \cos \left(\dfrac{A+B}{2}\right)}$ ${\cos A+\cos B=-2\sin \left(\dfrac{A+B}{2}\right) \sin \left(\dfrac{A-B}{2}\right)}$ ${\cos A-\cos B=2\cos \left(\dfrac{A+B}{2}\right) \cos \left(\dfrac{A-B}{2}\right)}$ These identities are applicable for all the triangles, not just for the right triangles. If A, B, and C are the vertices of a given triangle and a, b, and c are its corresponding sides, then: ${\dfrac{\sin A}{a}=\dfrac{\sin B}{b}=\dfrac{\sin C}{C}}$ Or ${\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{C}{\sin C}}$ ${\cos A=\dfrac{b^{2}+c^{2}-a^{2}}{2bc}}$ Or ${\cos B=\dfrac{a^{2}+c^{2}-b^{2}}{2ac}}$ Or ${\cos C=\dfrac{a^{2}+b^{2}-c^{2}}{2ab}}$ ${\dfrac{a-b}{a+b}=\dfrac{\tan \left(\dfrac{A-B}{2}\right) }{\tan \left(\dfrac{A+B}{2}\right) }}$ Or ${\dfrac{b-c}{b+c}=\dfrac{\tan \left(\dfrac{B-C}{2}\right) }{\tan \left(\dfrac{B+C}{2}\right) }}$ Or ${\dfrac{c-a}{c+a}=\dfrac{\tan \left(\dfrac{C-A}{2}\right) }{\tan \left(\dfrac{C+A}{2}\right) }}$ Simplify the expression ${\dfrac{\sin ^{2}\theta }{1-\cos \theta }+\dfrac{\sin ^{2}\theta }{1+\cos \theta }}$Solution:Given, ${\dfrac{\sin ^{2}\theta }{1-\cos \theta }+\dfrac{\sin ^{2}\theta }{1+\cos \theta }}$= ${\dfrac{\sin ^{2}\theta \left(1+\cos \theta \right) +\sin ^{2}\theta \left(1-\cos \theta \right) }{\left(1+\cos \theta \right) \left(1-\cos \theta \right) }}$= ${\dfrac{\sin ^{2}\theta \left(1+\cos \theta +1-\cos \theta \right) }{1-\cos ^{2}\theta }}$As we know from the Pythagorean identities, 1 – cos2 θ = sin2 θ= ${\dfrac{2\sin ^{2}\theta }{\sin ^{2}\theta }}$= ${2}$ Prove the given identity:${\dfrac{\sin ^{3}\theta +\cos ^{3}\theta }{\sin \theta +\cos \theta }+\sin \theta \cos \theta =1}$Solution:As we know, the sum of cubes formula: a3 + b3 = (a + b)(a2 – ab + b2)For a = sin θ and b = cos θ, we getsin3 θ + cos3 θ = (sin θ + cos θ)(sin2 θ – sin θ cos θ + cos2 θ) .....(i)Given, ${\dfrac{\sin ^{3}\theta +\cos ^{3}\theta }{\sin \theta +\cos \theta }+\sin \theta \cos \theta }$By using (i), we get= sin2 θ – sin θ cos θ + cos2 θ + sin θ cos θAs we know from the Pythagorean identities, sin2 θ + cos2 θ = 1= 1 – sin θ cos θ + sin θ cos θ= 1Thus, the given identity is proved. Prove the identity:(sec θ + cosec θ)2 – (tan2 θ + cot2 θ) = 2(1 + sec θ cosec θ)Solution:Given, (sec θ + cosec θ)2 – (tan2 θ + cot2 θ) = 2(1 + sec θ cosec θ)Expanding the left-hand-side,(sec θ + cosec θ)2 – (tan2 θ + cot2 θ)= sec2 θ + cosec2 θ + 2sec θ cosec θ – tan2 θ – cot2 θAs we know from the Pythagorean identities, sec2 θ = 1 + tan2 θ and cosec2 θ = 1 + cot2 θ= 1 + tan2 θ + 1 + cot2 θ + 2sec θ cosec θ – tan2 θ – cot2 θ= 1 + 1 + 2sec θ cosec θ= 2 + 2sec θ cosec θ= 2(1 + sec θ cosec θ)Thus, the given identity is proved. Last modified on March 7th, 2025 Share — copy and redistribute the material in any medium or format for any purpose, even commercially. Adapt — remix, transform, and build upon the material for any purpose, even commercially. The licensor cannot revoke these freedoms as long as you follow the license terms. Attribution — You must give appropriate credit , provide a link to the license, and indicate if changes were made . You may do so in any reasonable manner, but not in any way that suggests the licensor endorses you or your use. ShareAlike — If you remix, transform, or build upon the material, you must distribute your contributions under the same license as the original. No additional restrictions — You may not apply legal terms or technological measures that legally restrict others from doing anything the license permits. You do not have to comply with the license for elements of the material in the public domain or where your use is permitted by an applicable exception or limitation . No warranties are given. The license may not give you all of the permissions necessary for your intended use. For example, other rights such as publicity, privacy, or moral rights may limit how you use the material. Trigonometry Outline History Usage Functions (sin, cos, tan, inverse) Generalized trigonometry Reference Identities Exact constants Tables Unit circle Laws and theorems Sines Cosines Tangents Cotangents Pythagorean theorem Calculus Trigonometric substitution Integrals (inverse functions) Derivatives Trigonometric series Mathematicians Hipparchus Ptolemy Brahmagupta al-Hasib al-Battani Regiomontanus Viète de Moivre Euler Fourier vte In trigonometry, trigonometric identities are equalities that involve trigonometric functions and are true for every value of the occurring variables for which both sides of the equality are defined. Geometrically, these are identities involving certain functions of one or more angles. They are distinct from triangle identities, which are identities potentially involving angles but also involving side lengths or other lengths of a triangle. These identities are useful whenever expressions involving trigonometric functions need to be simplified. An important application is the integration of non-trigonometric functions: a common technique involves first using the substitution rule with a trigonometric function, and then simplifying the resulting integral with a trigonometric identity. Main article: Pythagorean trigonometric identity Trigonometric functions and their reciprocals on the unit circle. All of the right-angled triangles are similar, i.e. the ratios between their corresponding sides are the same. For sin, cos and tan the unit-length radius forms the hypotenuse of the triangle that defines them. The reciprocal identities arise as ratios of sides in the triangles where this unit line is no longer the hypotenuse. The triangle shaded blue illustrates the identity 1 + cot 2 θ = csc 2 θ {\displaystyle 1+\cot ^{2}\theta =\csc ^{2}\theta } , and the red triangle shows that tan 2 θ + 1 = sec 2 θ {\displaystyle \tan ^{2}\theta +1=\sec ^{2}\theta } . The basic relationship between the sine and cosine is given by the Pythagorean identity: sin 2 θ + cos 2 θ = 1 , {\displaystyle \sin ^{2}\theta +\cos ^{2}\theta =1,} where sin 2 θ {\displaystyle \sin ^{2}\theta } means (sin θ) 2 {\displaystyle {(\sin \theta)}^{2}} and cos 2 θ {\displaystyle \cos ^{2}\theta } means (cos θ) 2 . {\displaystyle {(\cos \theta)}^{2}.} This can be viewed as a version of the Pythagorean theorem, and follows from the equation x 2 + y 2 = 1 {\displaystyle x^{2}+y^{2}=1} for the unit circle. This equation can be solved for either the sine or the cosine: sin θ = ± 1 − cos 2 θ , cos θ = ± 1 − sin 2 θ . {\displaystyle {\begin{aligned}\sin \theta &=\pm {\sqrt {1-\cos ^{2}\theta }},\\\cos \theta &=\pm {\sqrt {1-\sin ^{2}\theta }}.\end{aligned}}} where the sign depends on the quadrant of θ . {\displaystyle \theta .} Dividing this identity by sin 2 θ {\displaystyle \sin ^{2}\theta } , cos 2 θ {\displaystyle \cos ^{2}\theta } , or both yields the following identities: 1 + cot 2 θ = csc 2 θ 1 + tan 2 θ = sec 2 θ sec 2 θ + csc 2 θ = sec 2 θ csc 2 θ {\displaystyle {\begin{aligned}&1+\cot ^{2}\theta =\csc ^{2}\theta \\&1+\tan ^{2}\theta =\sec ^{2}\theta \\&\sec ^{2}\theta +\csc ^{2}\theta =\sec ^{2}\theta \csc ^{2}\theta \end{aligned}}} Using these identities, it is possible to express any trigonometric function in terms of any other (up to a plus or minus sign): Each trigonometric function in terms of each of the other five.[1] in terms of sin θ {\displaystyle \sin \theta } csc θ {\displaystyle \csc \theta } cos θ {\displaystyle \cos \theta } sec θ {\displaystyle \sec \theta } tan θ {\displaystyle \tan \theta } cot θ {\displaystyle \cot \theta } sin θ = {\displaystyle \sin \theta =} sin θ {\displaystyle \sin \theta } 1 csc θ {\displaystyle {\frac {1}{\csc \theta }}} ± 1 − cos 2 θ {\displaystyle \pm {\sqrt {1-\cos ^{2}\theta }}} ± sec 2 θ − 1 sec θ {\displaystyle \pm {\frac {\sqrt {\sec ^{2}\theta -1}}{\sec \theta }}} ± tan θ 1 + tan 2 θ {\displaystyle \pm {\frac {\tan \theta }{\sqrt {1+\tan ^{2}\theta }}}} ± 1 1 + cot 2 θ {\displaystyle \pm {\frac {1}{\sqrt {1+\cot ^{2}\theta }}}} csc θ = {\displaystyle \csc \theta =} 1 sin θ {\displaystyle {\frac {1}{\sin \theta }}} csc θ {\displaystyle \csc \theta } ± 1 1 − cos 2 θ {\displaystyle \pm {\frac {1}{\sqrt {1-\cos ^{2}\theta }}}} ± sec θ sec 2 θ − 1 {\displaystyle \pm {\frac {\sec \theta }{\sqrt {\sec ^{2}\theta -1}}}} ± 1 + tan 2 θ tan θ {\displaystyle \pm {\frac {\sqrt {1+\tan ^{2}\theta }}{\tan \theta }}} ± 1 + cot 2 θ {\displaystyle \pm {\sqrt {1+\cot ^{2}\theta }}} cos θ = {\displaystyle \cos \theta =} ± 1 − sin 2 θ {\displaystyle \pm {\sqrt {1-\sin ^{2}\theta }}} ± csc 2 θ − 1 csc θ {\displaystyle \pm {\frac {\sqrt {\csc ^{2}\theta -1}}{\csc \theta }}} cos θ {\displaystyle \cos \theta } 1 sec θ {\displaystyle {\frac {1}{\sec \theta }}} ± 1 1 + tan 2 θ {\displaystyle \pm {\frac {1}{\sqrt {1+\tan ^{2}\theta }}}} ± cot θ 1 + cot 2 θ {\displaystyle \pm {\frac {\cot \theta }{\sqrt {1+\cot ^{2}\theta }}}} sec θ = {\displaystyle \sec \theta =} ± 1 1 − sin 2 θ {\displaystyle \pm {\frac {1}{\sqrt {1-\sin ^{2}\theta }}}} ± csc θ csc 2 θ − 1 {\displaystyle \pm {\frac {\csc \theta }{\sqrt {\csc ^{2}\theta -1}}}} 1 cos θ {\displaystyle {\frac {1}{\cos \theta }}} sec θ {\displaystyle \sec \theta } ± 1 + tan 2 θ {\displaystyle \pm {\sqrt {1+\tan ^{2}\theta }}} ± 1 + cot 2 θ cot θ {\displaystyle \pm {\frac {\sqrt {1+\cot ^{2}\theta }}{\cot \theta }}} ± sec 2 θ − 1 {\displaystyle \pm {\sqrt {\sec ^{2}\theta -1}}} tan θ = {\displaystyle \tan \theta =} ± sin θ 1 − sin 2 θ {\displaystyle \pm {\frac {\sin \theta }{\sqrt {1-\sin ^{2}\theta }}}} ± csc 2 θ − 1 {\displaystyle \pm {\frac {\sqrt {\csc ^{2}\theta -1}}{\csc \theta }}} ± cos 2 θ cos θ {\displaystyle \pm {\frac {\sqrt {1-\cos ^{2}\theta }}{\cos \theta }}} ± sec 2 θ − 1 {\displaystyle \pm {\sqrt {\sec ^{2}\theta -1}}} tan θ {\displaystyle \tan \theta } 1 cot θ {\displaystyle {\frac {1}{\cot \theta }}} cot θ = {\displaystyle \cot \theta =} ± 1 − sin 2 θ sin θ {\displaystyle \pm {\frac {\sin \theta }{\sqrt {1-\sin ^{2}\theta }}}} ± 1 − cos 2 θ {\displaystyle \pm {\sqrt {1-\cos ^{2}\theta }}} ± sec 2 θ − 1 {\displaystyle \pm {\frac {\sqrt {\sec ^{2}\theta -1}}{\sec \theta }}} 1 tan θ {\displaystyle {\frac {1}{\tan \theta }}} cot θ {\displaystyle \cot \theta } By examining the unit circle, one can establish the following properties of the trigonometric functions. Transformation of coordinates (a,b) when shifting the reflection angle α {\displaystyle \alpha } in increments of π 4 {\displaystyle {\frac {\pi }{4}}} When the direction of a Euclidean vector is represented by an angle θ , {\displaystyle \theta ,} this is the angle determined by the free vector (starting at the origin) and the positive x {\displaystyle x} -unit vector. The same concept may also be applied to lines in an Euclidean space, where the angle is that determined by a parallel to the given line through the origin and the positive x {\displaystyle x} -axis. If a line (vector) with direction θ {\displaystyle \theta } is reflected about a line with direction α , {\displaystyle \alpha ,} then the direction angle θ ′ {\displaystyle \theta ^{\prime }} of this reflected line (vector) has the value θ ′ = 2 α − θ . {\displaystyle \theta ^{\prime }=2\alpha -\theta .} The values of the trigonometric functions of these angles θ , θ ′ {\displaystyle \theta ,\;\theta ^{\prime }} for specific angles α {\displaystyle \alpha } satisfy simple identities: either they are equal, or have opposite signs, or employ the complementary trigonometric function. These are also known as reduction formulae.[2] θ {\displaystyle \theta } reflected in α = 0 {\displaystyle \alpha =0} [3]odd/even identities θ {\displaystyle \theta } reflected in α = π 4 {\displaystyle \alpha ={\frac {\pi }{4}}} θ {\displaystyle \theta } reflected in α = π 2 {\displaystyle \alpha ={\frac {\pi }{2}}} θ {\displaystyle \theta } reflected in α = 3 π 4 {\displaystyle \alpha ={\frac {3\pi }{4}}} θ {\displaystyle \theta } reflected in α = π {\displaystyle \alpha =\pi } compare to α = 0 {\displaystyle \alpha =0} sin (− θ) = − sin θ {\displaystyle \sin(-\theta)=-\sin \theta } sin (π 2 − θ) = cos θ {\displaystyle \sin \left({\tfrac {\pi }{2}}-\theta \right)=\cos \theta } sin (π − θ) = + sin θ {\displaystyle \sin(\pi -\theta)=+\sin \theta } sin (3 π 2 − θ) = − cos θ {\displaystyle \sin \left({\tfrac {3\pi }{2}}-\theta \right)=-\cos \theta } sin (2 π − θ) = − sin (θ) = sin (− θ) {\displaystyle \sin(2\pi -\theta)=-\sin(\theta)=\sin(-\theta)} cos (− θ) = + cos θ {\displaystyle \cos(-\theta)=+\cos \theta } cos (π 2 − θ) = sin θ {\displaystyle \cos \left({\tfrac {\pi }{2}}-\theta \right)=\sin \theta } cos (π − θ) = − cos θ {\displaystyle \cos(\pi -\theta)=-\cos \theta } cos (3 π 2 − θ) = − sin θ {\displaystyle \cos \left({\tfrac {3\pi }{2}}-\theta \right)=-\sin \theta } cos (2 π − θ) = + cos (θ) = cos (− θ) {\displaystyle \cos(2\pi -\theta)=+\cos(\theta)=\cos(-\theta)} tan (− θ) = − tan θ {\displaystyle \tan(-\theta)=-\tan \theta } tan (π 2 − θ) = cot θ {\displaystyle \tan \left({\tfrac {\pi }{2}}-\theta \right)=\cot \theta } tan (π − θ) = − tan θ {\displaystyle \tan(\pi -\theta)=-\tan \theta } tan (3 π 2 − θ) = + cot θ {\displaystyle \tan \left({\tfrac {3\pi }{2}}-\theta \right)=+\cot \theta } tan (2 π − θ) = − tan (θ) = tan (− θ) {\displaystyle \tan(2\pi -\theta)=-\tan(\theta)=\tan(-\theta)} csc (− θ) = − csc θ {\displaystyle \csc(-\theta)=-\csc \theta } csc (π 2 − θ) = sec θ {\displaystyle \csc \left({\tfrac {\pi }{2}}-\theta \right)=\sec \theta } csc (π − θ) = + csc θ {\displaystyle \csc(\pi -\theta)=+\csc \theta } csc (3 π 2 − θ) = − sec θ {\displaystyle \csc \left({\tfrac {3\pi }{2}}-\theta \right)=-\sec \theta } csc (2 π − θ) = − csc (θ) = csc (− θ) {\displaystyle \csc(2\pi -\theta)=-\csc(\theta)=\csc(-\theta)} sec (− θ) = + sec θ {\displaystyle \sec(-\theta)=+\sec \theta } sec (π 2 − θ) = csc θ {\displaystyle \sec \left({\tfrac {\pi }{2}}-\theta \right)=\csc \theta } sec (π − θ) = − sec θ {\displaystyle \sec(\pi -\theta)=-\sec \theta } sec (3 π 2 − θ) = − csc θ {\displaystyle \sec \left({\tfrac {3\pi }{2}}-\theta \right)=-\csc \theta } sec (2 π − θ) = + sec (θ) = sec (− θ) {\displaystyle \sec(2\pi -\theta)=+\sec(\theta)=\sec(-\theta)} cot (− θ) = − cot θ {\displaystyle \cot(-\theta)=-\cot \theta } cot (π 2 − θ) = tan θ {\displaystyle \cot \left({\tfrac {\pi }{2}}-\theta \right)=\tan \theta } cot (π − θ) = − cot θ {\displaystyle \cot(\pi -\theta)=-\cot \theta } cot (3 π 2 − θ) = + tan θ {\displaystyle \cot \left({\tfrac {3\pi }{2}}-\theta \right)=+\tan \theta } cot (2 π − θ) = − cot (θ) = cot (− θ) {\displaystyle \cot(2\pi -\theta)=-\cot(\theta)=\cot(-\theta)} Transformation of coordinates (a,b) when shifting the angle θ {\displaystyle \theta } in increments of π 2 {\displaystyle {\frac {\pi }{2}}} Shift by one quarter period Shift by one half period Shift by full periods[4] Period sin (θ ± π 2) = ± cos θ {\displaystyle \sin(\theta \pm {\tfrac {\pi }{2}})=\pm \cos \theta } sin (θ + π) = − sin θ {\displaystyle \sin(\theta +\pi)=-\sin \theta } sin (θ + k · 2 π) = + sin θ {\displaystyle \sin(\theta +k\cdot 2\pi)=+\sin \theta } 2 π {\displaystyle 2\pi } cos (θ ± π 2) = ∓ sin θ {\displaystyle \cos(\theta \pm {\tfrac {\pi }{2}})=\mp \sin \theta } cos (θ + π) = − cos θ {\displaystyle \cos(\theta +\pi)=-\cos \theta } cos (θ + k · 2 π) = + cos θ {\displaystyle \cos(\theta +k\cdot 2\pi)=+\cos \theta } 2 π {\displaystyle 2\pi } csc (θ ± π 2) = ± sec θ {\displaystyle \csc(\theta \pm {\tfrac {\pi }{2}})=\mp \sec \theta } csc (θ + π) = − csc θ {\displaystyle \csc(\theta +\pi)=-\csc \theta } csc (θ + k · 2 π) = + csc θ {\displaystyle \csc(\theta +k\cdot 2\pi)=+\csc \theta } 2 π {\displaystyle 2\pi } sec (θ ± π 2) = ∓ csc θ {\displaystyle \sec(\theta \pm {\tfrac {\pi }{2}})=\mp \csc \theta } sec (θ + π) = − sec θ {\displaystyle \sec(\theta +\pi)=-\sec \theta } sec (θ + k · 2 π) = + sec θ {\displaystyle \sec(\theta +k\cdot 2\pi)=+\sec \theta } 2 π {\displaystyle 2\pi } tan (θ ± π 4) = tan θ ± 1 1 ∓ tan θ {\displaystyle \tan(\theta \pm {\tfrac {\pi }{4}})={\tfrac {\tan \theta \pm 1}{1\mp \tan \theta }}} tan (θ + π 2) = − cot θ {\displaystyle \tan(\theta +{\tfrac {\pi }{2}})=-\cot \theta } tan (θ + k · π) = + tan θ {\displaystyle \tan(\theta +k\cdot \pi)=+\tan \theta } π {\displaystyle \pi } cot (θ ± π 4) = cot θ ∓ 1 1 ± cot θ {\displaystyle \cot(\theta \pm {\tfrac {\pi }{4}})={\tfrac {\cot \theta \mp 1}{1\pm \cot \theta }}} cot (θ + π 2) = − tan θ {\displaystyle \cot(\theta +{\tfrac {\pi }{2}})=-\tan \theta } cot (θ + k · π) = + cot θ {\displaystyle \cot(\theta +k\cdot \pi)=+\cot \theta } π {\displaystyle \pi } The sign of trigonometric functions depends on quadrant of the angle. If − π < θ ≤ π {\displaystyle {-\pi }