I'm not a bot

7 unsolved math problems

The 2025 Clay Research Conference will held on Wednesday and Thursday, 1-2 October. Associated workshops will be held Monday, Tuesday, and Friday during the week of the conference. The Seven Millennium Prize Problems are the most well-known and important unsolved problems in mathematics. A private nonprofit foundation Clay Mathematics Institute that is devoted to mathematical research, famously challenged the mathematical community in the year 2000 to solve these unique seven problems, and a sum of US $1,000,000 reward was established for the solvers of each of the seven problems. Out of the seven Millennium prize problems, one of the problems has been solved, and the other six are a great deal of current research.With the spin of the century, the timing of the announcement of the Millennium Prize Problems was a homage to a famous speech of the famous David Hilbert to the International Congress of Mathematicians in the year 1900 in the city of Paris. The 23 unsolved problems that were posed by Hilbert were studied by countless 20th century mathematicians, which led not only to solutions to some of these difficult problems but it also led to the development of new ideas as well as new research topics. There are some of Hilbert's problems that still remain open-- namely the famous Riemann hypothesis. These seven problems encompass a diverse group of topics, which include theoretical computer science as well as physics, as well as topics of pure mathematical areas such as number theory, algebraic geometry, as well as topics of topology.7 Millennium Prize Problems1. Yang-Mills and Mass GapComputer simulations as well as various experiments suggest the existence of a "mass gap" in the solution to the quantum versions of the Yang-Mills equations. But no proof of this property is known. A Yang-Mills theory is known to be a theory in quantum physics that is a generalization of Maxwell's work on electromagnetic forces to the strong as well as weak nuclear forces. It is a key ingredient in the Standard Model of particle physics. This Standard Model is said to provide a framework for explaining electromagnetic as well as providing nuclear forces and also classifying subatomic particles. In particular, successful applications of the theory to experiments as well as simplified models have involved a "mass gap," which can be formally defined as the difference between the default energy in a vaccum as well as also the energy in the next lowest energy state. So this quantity is also known as the mass of the lightest particle in the theory. A solution to the Millennium Problem will include both a set of formal axioms that characterize the theory as well as will show that it is internally logically consistent.2. Riemann HypothesisThe prime number theorem determines the average distribution of the prime numbers. Whereas the Riemann hypothesis basically describes the deviation from the average. It was formulated in Riemann's 1859 paper, which asserts that all the 'non-obvious' zeros of the zeta function are complex numbers with real part 1/2.3. P vs NP ProblemIf it is easy to check that a solution to a problem is right, can you say that it is also easy to solve the problem? This is said to be the exact essence of the NP question vs P question. Typical of the NP problems is that of the Hamiltonian Path Problem: let’s suppose given N cities to visit, how can one do this without visiting a city twice? If you give me a solution, I can easily check that the problem is correct, but it is difficult to find a solution.4. Navier–Stokes EquationThe Navier-Stokes equation is said to be the equation that governs the flow of fluids such as water as well as air. However, there is no proof for the most basic questions one can ask: do solutions exist as well as are they unique? Why ask for proof? Because proof gives not only certitude but proof also gives understanding.5. Hodge ConjectureHodge conjecture, the answer to this determines how much of the topology of the solution set of a system of algebraic equations can be defined in terms of further algebraic equations. The Hodge conjecture comes in picture in certain special cases, for example, when the solution set has a dimension less than four. But in dimension four it is unknown.This conjecture is also known to be a statement about geometric shapes cut out by polynomial equations over complex numbers. These are also known as complex algebraic varieties. An extremely useful tool in the study of these varieties was the construction of groups which is also known as cohomology groups, which contained information about the structure of the varieties. 6. Poincaré ConjectureThe French mathematician Henri Poincaré in the year 1904. He was the one who asked if the three-dimensional sphere is characterized as the unique simply connected three-manifold. The Poincaré conjecture is known as a special case of Thurston's geometrization conjecture. This Poincaré conjecture proof tells us that every three-manifold is built from a set of standard pieces, each with one of eight well-understood geometries.7. Birch and Swinnerton-Dyer ConjectureThis z conjecture is basically supported by much experimental evidence that relates the number of points on an elliptic curve mod p to the rank of the group of rational points. The Millennium Problems are a set of seven problems for which the Clay Mathematics Institute offered a US $7 million prize fund ($1 million per problem) to celebrate the new millennium in May 2000. The problems all have significant impacts on their field of mathematics and beyond, and were all unsolved at the time of the offering of the prize. The seven problems are the Birch and Swinnerton-Dyer Conjecture, the Hodge Conjecture, the Navier-Stokes Equations, P versus NP, the Poincaré Conjecture, the Riemann Hypothesis, and the Yang-Mills Theory. In 2003, the Poincaré Conjecture was proven by Russian mathematician Grigori Perelman. History Announcement The millennium problems were first announced at Millennium Meeting on May 24, 2000 at the Collège de France. Timothy Gowers first presented a lecture titled The Importance of Mathematics as an introduction. After this, the British mathematician Michael Atiyah and the American John Tate announced the prize: one million dollars to anyone who could solve one of the seven most difficult open problems at the time. A small committee of mathematicians, selected by the scientific advisory board (SAB) of the Clay Mathematical Institute (which also had organized the meeting), had selected the problems over the previous several months. They were led by Arthur Jaffe, the first director of the CMI, the former director of the American Mathematical Society, and the incumbent of the Landon Clay Chair in Mathematics at Harvard University. This committee included such luminaries as Andrew Wiles, the aforementioned Atiyah and Tate, the American Edward Twitten, and the French Alaine Connes. Motivation Partly, the motive of the CMI and its founder (see "Rules and Financing") was the founder's support of mathematical research. However, specifically, the inspiration was a similar prize exactly a hundred years earlier. Paris had seen a similar event then, at the second International Congress of Mathematicians. The famous German mathematician David Hilbert drew up a list of 23 "Hilbert Problems" on August 8, "setting the agenda for the twentieth century". (Devlin 2003, pp. 2–3) These problems he believed to be the most significant and important unsolved in mathematics. Some of these problems were either shown to be unsolvable, indefinite, or trivial. However, many were difficult problems, and enormous prestige was given to a mathematician who solved one of them as soon as the mathematical community had pronounced his solution correct. All but one of these problems had been solved by the meeting in 2000, and thus it was natural to create a new set of such problems. Wiles, however, notes that Hilbert's and the CMI's motivations were slightly different; "Hilbert was trying to guide mathematics by his problems; we're trying to record great unsolved problems. There are big problems in mathematics that are important but where it is very hard to isolate one problem that captures the program." (Devlin 2003, p. 3) Solving of the Poincaré Conjecture On April 7–11, 2003, Russian mathematician Grigori Perelman, a member of the Steklov Institute of Mathematics, a division of the Russian Academy of Sciences in St. Petersburg, presented his proof of the Poincaré Conjecture during the Simons Lecture Series at the MIT Mathematics Department. He gave three lectures, titled "Ricci Flow and Geometrization of Three-Manifolds", on April 7, 9, and 11. These were his first public presentation of the important results he had published earlier, in November 2002 and March 2003. Perelman's paper proved not only the Poincaré Conjecture, but a generalization known as Thurston's Geometrization Conjecture. The former merely stated that every closed, simply-connected three manifold is homeomorphic to the three sphere; a sphere with a three-dimensional surface, or a four-dimensional sphere. Thurston's conjecture extended the conjecture to any positive integer n, stating that a compact n manifold is homotopy equivalent to the n sphere iff it is homeomorphic to the n sphere, or, more simply, that only a n manifold with no holes was simply connected. All cases of this conjecture had been proven up to this point except the case where . In 1995, Perelman had learned of Ricci flow, the key to his proof, from Hamilton in the United States. Returning to Russia, though publishing few results, he became an expert on Ricci flow and differential geometry in general. The aforementioned November 2002 and March 2003 publications were then posted on the internet, climaxing to Perelman's lectures in April of that year. Previous proofs had been purported, most notably Dunwoody's, a year prior to Perelman's, but they had all proved false. Perelman's, however, was well received by the mathematical community, and the conjecture was declared proven in 2006 after four years of debate and three 300-page-long publications filling in details of the proof. Perelman, however, declined the Fields Medal offered to him by the IMU. He also retired from mathematics, citing in a New Yorker interview his colleagues' unethical actions, particularly the credit claimed by the authors of the third 300-page-long publication. Controversies regarding the third publication have abounded since then, resulting in the collapse of a planned January 2007 Poincaré Conjecture "All-Stars" meeting. (References: MathWorld article, ScienceMag article) Rules and Financing The prize fund is financed privately by Landon Clay, the man who had established the CMI one year earlier as a nonprofit organization based in his hometown; Cambridge, MA to aid mathematical research. Clay, a well-off mutual-fund manager, though not a mathematician, was greatly interested in and supportive of the mathematical community. The rules were laid out as follows: The SAB would consider a proof of one of the problems on several conditions. Firstly, the proof had to be complete. Secondly, it had to "be published in a refereed mathematics publication of worldwide repute and […] general acceptance in the mathematics community two years after". (Millennium Prize Rules; as of January 19, 2005) If these conditions are met, the SAB will appoint an advisory committee to thoroughly examine the solution. This committee would consist of at least two world-renowned mathematicians and at least one member of the SAB. After the analysis, the committee would report back to the SAB. The SAB would then report to the directors of the CMI, possibly giving recommendations of whether or not the prize should be awarded and (in the case of a group of mathematicians collaborating on one problem or multiple mathematicians solving a problem near-simultaneously) which person(s) should receive the prize. In the case of multiple prizewinners, the prize would be divided proportionally according to the judgement of the directors. Counterexamples are put through essentially the same process; again, the directors make the final decision, though the SAB can advise them. In the special case of a problem being shown to be false as stated, but ambiguous with a small adjustment, a small prize may be awarded to the mathematician who discovered this; though the money would be taken from funds other than the Millennium Prize ones. The new problem would then replace the old in the selection of problems, with the same process and conditions. Problems Birch and Swinnerton-Dyer Conjecture Main article: Birch and Swinnerton-Dyer Conjecture The Birch and Swinnerton-Dyer conjecture relates the rank of the abelian group of points over a number field of an elliptic curve to the order of the zero of the associated -function at . As of 2005, it has been proved only in special cases, such as over certain quadratic fields (by Henri Darmon of McGill University). It has been an open problem for around 40 years, and has stimulated much research; its status as one of the most challenging mathematical questions has become widely recognized. Hodge Conjecture Main article: Hodge Conjecture The Hodge conjecture asserts that structures known as Hodge classes, which can be elementarily described as geometric representations of a given manifold's topological properties, are composed of algebraic cycles. More rigorously, the common phrasing for the conjecture is "Given a projective complex manifold, every Hodge class on it is a linear combination with rational coefficients of the cohomology classes of complex subvarieties of it." Main article: Navier-Stokes Equations The Navier-Stokes equations describe the motion of fluids. These equations establish that the acceleration of fluid particles is simply the product of changes in pressure and dissipative viscous forces (similar to friction) acting inside the fluid. These viscous forces originate in molecular interactions and dictate how sticky (viscous) a fluid is. Thus, the Navier-Stokes equations are a dynamical statement of the balance of forces acting at any given region of the fluid. P versus NP Main article: P versus NP The problem of P versus NP is an important problem in computability and complexity theory relating to whether decision problems (problems admitting a yes or no answer) whose solutions can be verified in polynomial time (as a function of the input, often expressed using big-O notation) can also be solved in polynomial time. The set consists of decision problems such that there exists a deterministic computer program (or Turing machine) that decides in polynomial time. The set , informally, consists of decision problems whose "yes" instances can be verified by a deterministic program in polynomial time, given a certificate. Whether unknown, though many problems can be shown to be NP-complete - that is, if a problem is NP-complete, then any NP problem can be reduced to in polynomial time. This implies that if any NP-complete problem has a polynomial time solution, then . Poincaré Conjecture Main article: Poincaré Conjecture In elementary terms, the Poincaré conjecture states that the only three-manifold with no "holes" is the three-sphere. This would also show that the only -manifold with no "holes" is the -sphere; the case is trivial, the case is a classic problem, and the truth of the statement for was verified by Stephen Smale in 1961. More rigorously, the conjecture is expressed as "Every simply connected, compact three-manifold (without boundary) is homeomorphic to the three-sphere." Riemann Hypothesis Main article: Riemann Hypothesis The Riemann hypothesis is a well-known conjecture in analytic number theory that states that all nontrivial (the trivial roots are when) zeros of the Riemann zeta function have real part . The Riemann Hypothesis is an important problem in the study of prime numbers. Let denote the number of primes less than or equal to x, and let . Then an equivalent statement of the Riemann hypothesis is that . Yang-Mills Theory Main article: Yang-Mills Theory The quantum Yang-Mills theory (no quarks) with a non-abelian gauge group is an exception to the general rule that nontrivial (i.e. interacting) quantum field theories that we know of in 4D are effective field theories with a cutoff scale. It has a property known as asymptotic freedom, meaning that it has a trivial UV fixed point. Because of this, this is the simplest model to pin our hopes on for a nontrivial constructive QFT model in 4D. (QCD, with its fermionic quarks is obviously more complicated). It has already been well proven at the standards of theoretical physics, but not mathematical physics, that the quantum Yang–Mills theory for a non-abelian Lie group exhibits a property known as confinement. See Also References Devlin, Keith J (2003). Basic Books. The Millennium Problems. ISBN 978-0465017300. External Links Seven mathematical problems with a US$1 million prize for each solution This article is about the math prizes. For the technology prize, see Millennium Technology Prize. Millennium Prize Problems Birch and Swinnerton-Dyer conjecture Hodge conjecture Navier–Stokes existence and smoothness P versus NP problem Poincaré conjecture (solved) Riemann hypothesis Yang–Mills existence and mass gap vte The Millennium Prize Problems are seven well-known complex mathematical problems selected by the Clay Mathematics Institute in 2000. The Clay Institute has pledged a US $1 million prize for the first correct solution to each problem. The Clay Mathematics Institute officially designated the title Millennium Problem for the seven unsolved mathematical problems, the Birch and Swinnerton-Dyer conjecture, Hodge conjecture, Navier–Stokes existence and smoothness, P versus NP problem, Riemann hypothesis, Yang–Mills existence and mass gap, and the Poincaré conjecture at the Millennium Meeting held on May 24, 2000. Thus, on the official website of the Clay Mathematics Institute, these seven problems are officially called the Millennium Problems. To date, the only Millennium Prize problem to have been solved is the Poincaré conjecture. The Clay Institute awarded the monetary prize to Russian mathematician Grigori Perelman in 2010. However, he declined the award as it was not also offered to Richard S. Hamilton, upon whose work Perelman built.[1] The Clay Institute was inspired by a set of twenty-three problems organized by the mathematician David Hilbert in 1900 which were highly influential in driving the progress of mathematics in the twentieth century.[2] The seven selected problems span a number of mathematical fields, namely algebraic geometry, arithmetic geometry, geometric topology, mathematical physics, number theory, partial differential equations, and theoretical computer science. Unlike Hilbert's problems, the problems selected by the Clay Institute were already renowned among professional mathematicians, with many actively working towards their resolution.[3] The seven problems were officially announced by John Tate and Michael Atiyah during a ceremony held on May 24, 2000 (at the amphithéâtre Marguerite de Navarre) in the Collège de France in Paris.[4] Grigori Perelman, who had begun work on the Poincaré conjecture in the 1990s, released his proof in 2002 and 2003. His refusal of the Clay Institute's monetary prize in 2010 was widely covered in the media. The other six Millennium Prize Problems remain unsolved, despite a large number of unsatisfactory proofs by both amateur and professional mathematicians. Andrew Wiles, as part of the Clay Institute's scientific advisory board, hoped that the choice of US$1 million prize money would popularize, among general audiences, both the selected problems as well as the "excitement of mathematical endeavor".[5] Another board member, Fields medalist Alain Connes, hoped that the publicity around the unsolved problems would help to combat the "wrong idea" among the public that mathematics would be "overtaken by computers".[6] Some mathematicians have been more critical. Anatoly Vershik characterized their monetary prize as "show business" representing the "worst manifestations of present-day mass culture", and thought that there are more meaningful ways to invest in public appreciation of mathematics.[7] He viewed the superficial media treatments of Perelman and his work, with disproportionate attention being placed on the prize value itself, as unsurprising. By contrast, Vershik praised the Clay Institute's direct funding of research conferences and young researchers. Vershik's comments were later echoed by Fields medalist Shing-Tung Yau, who was additionally critical of the idea of a foundation taking actions to "appropriate" fundamental mathematical questions and "attach its name to them".[8] Main article: Poincaré conjecture A compact 2-dimensional surface without boundary is topologically homeomorphic to a 2-sphere if every loop can be continuously tightened to a point. The Poincaré conjecture asserts that the same is true for 3-dimensional spaces. In the field of geometric topology, a two-dimensional sphere is characterized by the fact that it is the only closed and simply-connected two-dimensional surface. In 1904, Henri Poincaré posed the question of whether an analogous statement holds true for three-dimensional shapes. This came to be known as the Poincaré conjecture, the precise formulation of which states: Any three-dimensional topological manifold which is closed and simply-connected must be homeomorphic to the 3-sphere. Although the conjecture is usually stated in this form, it is equivalent (as was discovered in the 1950s) to pose it in the context of smooth manifolds and diffeomorphisms. A proof of this conjecture, together with the more powerful geometrization conjecture, was given by Grigori Perelman in 2002 and 2003. Perelman's solution completed Richard Hamilton's program for the solution of the geometrization conjecture, which he had developed over the course of the preceding twenty years. Hamilton and Perelman's work revolved around Hamilton's Ricci flow, which is a complicated system of partial differential equations defined in the field of Riemannian geometry. For his contributions to the theory of Ricci flow, Perelman was awarded the Fields Medal in 2006. However, he declined to accept the prize.[9] For his proof of the Poincaré conjecture, Perelman was awarded the Millennium Prize on March 18, 2010.[10] However, he declined the award and the associated prize money, stating that Hamilton's contribution was no less than his own.[1] Main article: Birch and Swinnerton-Dyer conjecture The Birch and Swinnerton-Dyer conjecture deals with certain types of equations: those defining elliptic curves over the rational numbers. The conjecture is that there is a simple way to tell whether such equations have a finite or infinite number of rational solutions. More specifically, the Millennium Prize version of the conjecture is that, if the elliptic curve E has rank r, then the L-function L(E, s) associated with it vanishes to order r at s = 1. Hilbert's tenth problem dealt with a more general type of equation, and in that case it was proven that there is no algorithmic way to decide whether a given equation even has any solutions. The official statement of the problem was given by Andrew Wiles.[11] Main article: Hodge conjecture The Hodge conjecture is that for projective algebraic varieties, Hodge cycles are rational linear combinations of algebraic cycles. Hdg k (X) = H 2 k (X , Q) ∩ H k , k (X) . {\displaystyle \operatorname {Hdg} ^{k}(X)=H^{2k}(X,\mathbb {Q})\cap H^{k,k}(X).} We call this the group of Hodge classes of degree 2k on X. The modern statement of the Hodge conjecture is: Let X be a non-singular complex projective variety. Then every Hodge class on X is a linear combination with rational coefficients of the cohomology classes of complex subvarieties of X. The official statement of the problem was given by Pierre Deligne.[12] Main article: Navier–Stokes existence and smoothness ∂ u ∂ t ⏟ Variation + (u ⋅ ∇) u ⏟ Convection ⏞ Inertia (per volume) − ν ∇ 2 u ⏟ Diffusion = − ∇ w ⏟ Internal source ⏞ Divergence of stress + g ⏟ External source . {\displaystyle \overbrace {\underbrace {\frac {\partial \mathbf {u} }{\partial t}} _{\begin{smallmatrix} {\text{Variation}}\end{smallmatrix}}+\underbrace {(\mathbf {u} \cdot abla)\mathbf {u} } _{\begin{smallmatrix}{\text{Convection}}\end{smallmatrix}}} ^{\text{Inertia (per volume)}}\overbrace {{}-\underbrace {u \,abla ^{2}\mathbf {u} } _{\text{Diffusion}}=\underbrace {-abla w} _{\begin{smallmatrix}{\text{Internal}}\\ {\text{source}}\end{smallmatrix}}} ^{\text{Divergence of stress}}+\underbrace {\mathbf {g} } _{\begin{smallmatrix}{\text{External}}\\{\text{source}}\end{smallmatrix}}.} The Navier–Stokes equations describe the motion of fluids, and are one of the pillars of fluid mechanics. However, theoretical understanding of their solutions is incomplete, despite its importance in science and engineering. For the three-dimensional system of equations, and given some initial conditions, mathematicians have not yet proven that smooth solutions always exist. This is called the Navier–Stokes existence and smoothness problem. The problem, restricted to the case of an incompressible flow, is to prove either that smooth, globally defined solutions exist that meet certain conditions, or that they do not always exist and the equations break down. The official statement of the problem was given by Charles Fefferman.[13] Euler diagram for P, NP, NP-complete, and NP-hard set of problems (excluding the empty language and its complement, which belong to P but are not NP-complete) Main article: P versus NP problem The question is whether or not, for all problems for which an algorithm can verify a given solution quickly (that is, in polynomial time), an algorithm can also find that solution quickly. Since the former describes the class of problems termed NP, while the latter describes P, the question is equivalent to asking whether all problems in NP are also in P. This is generally considered one of the most important open questions in mathematics and theoretical computer science as it has far-reaching consequences to other problems in mathematics, to biology,[14] philosophy[15] and to cryptography (see P versus NP problem proof consequences). A common example of an NP problem not known to be in P is the Boolean satisfiability problem. Most mathematicians and computer scientists expect that P ≠ NP; however, it remains unproven.[16] The official statement of the problem was given by Stephen Cook.[17] The real part (red) and imaginary part (blue) of the Riemann zeta function along the critical line Re(s) = 1/2. The first nontrivial zeros can be seen at Im(s) = ±14.135, ±21.022 and ±25.011. Main article: Riemann hypothesis ζ (s) = ∑ n = 1 ∞ n − s = 1 1 s + 1 2 s + 1 3 s + ⋯ {\displaystyle \zeta (s)=\sum _{n=1}^{\infty }n^{-s}={\frac {1}{1^{s}}}+{\frac {1}{2^{s}}}+{\frac {1}{3^{s}}}+\cdots } The Riemann zeta function ζ(s) is a function whose arguments may be any complex number other than 1, and whose values are also complex. Its analytical continuation has zeros at the negative even integers; that is, ζ(s) = 0 when s is one of −2, −4, −6, ... These are called its trivial zeros. However, the negative even integers are not the only values for which the zeta function is zero. The other ones are called nontrivial zeros. The Riemann hypothesis is concerned with the locations of these nontrivial zeros, and states that: The real part of every nontrivial zero of the Riemann zeta function is 1/2. The Riemann hypothesis is that all nontrivial zeros of the analytical continuation of the Riemann zeta function have a real part of 1/2. A proof or disproof of this would have far-reaching implications in number theory, especially for the distribution of prime numbers. This was Hilbert's eighth problem, and is still considered an important open problem a century later. The problem has been well-known ever since it was originally posed by Bernhard Riemann in 1860. The Clay Institute's exposition of the problem was given by Enrico Bombieri.[18] Main article: Yang–Mills existence and mass gap In quantum field theory, the mass gap is the difference in energy between the vacuum and the next lowest energy state. The energy of the vacuum is zero by definition, and assuming that all energy states can be thought of as particles in plane-waves, the mass gap is the mass of the lightest particle. For a given real field ϕ (x) {\displaystyle \phi (x)} , we can say that the theory has a mass gap if the two-point function has the property ⟨ ϕ (0 , t) ϕ (0 , 0) ⟩ ∼ ∑ n A n exp ⁡ (− Δ n t) {\displaystyle \langle \phi (0,t)\phi (0,0)\rangle \sim \sum _{n}A_{n}\exp \left(-\Delta _{n}t\right)} with Δ 0 > 0 {\displaystyle \Delta _{0}>0} being the lowest energy value in the spectrum of the Hamiltonian and thus the mass gap. This quantity, easy to generalize to other fields, is what is generally measured in lattice computations. Quantum Yang–Mills theory is the current grounding for the majority of theoretical applications of thought to the reality and potential realities of elementary particle physics.[19] The theory is a generalization of the Maxwell theory of electromagnetism where the chromo-electromagnetic field itself carries charge. As a classical field theory it has solutions which travel at the speed of light so that its quantum version should describe massless particles (gluons). However, the postulated phenomenon of color confinement permits only bound states of gluons, forming massive particles. This is the mass gap. Another aspect of confinement is asymptotic freedom which makes it conceivable that quantum Yang-Mills theory exists without restriction to low energy scales. The problem is to establish rigorously the existence of the quantum Yang–Mills theory and a mass gap. Prove that for any compact simple gauge group G, a non-trivial quantum Yang–Mills theory exists on R 4 {\displaystyle \mathbb {R} ^{4}} and has a mass gap Δ > 0. Existence includes establishing axiomatic properties at least as strong as those cited in Streater & Wightman (1964),[20] Osterwalder & Schrader (1973),[21] and Osterwalder & Schrader (1975).[22] The official statement of the problem was given by Arthur Jaffe and Edward Witten.[23] Mathematics portal Beal conjecture Hilbert's problems List of mathematics awards List of unsolved problems in mathematics Smale's problems Paul Wolfskehl

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In Carlson, James; Jaffe, Arthur; Wiles, Andrew (eds.). The millennium prize problems. Providence, RI: American Mathematical Society and Clay Mathematics Institute. pp. 129–152. ISBN 978-0-8218-3679-8. This article incorporates material from Millennium Problems on PlanetMath, which is licensed under the Creative Commons Attribution/Share-Alike License. Carlson, James; Jaffe, Arthur; Wiles, Andrew, eds. (2006). The Millennium Prize Problems. Providence, RI: American Mathematical Society and Clay Mathematics Institute. ISBN 978-0-8218-3679-8. Devlin, Keith J. (2003) [2002]. The Millennium Problems: The Seven Greatest Unsolved Mathematical Puzzles of Our Time. New York: Basic Books. ISBN 0-465-01729-0. Wikiquote has quotations related to Millennium Prize Problems. The Millennium Prize Problems Retrieved from " The only Millennium Problem that has been solved to date is the Poincare conjecture, a problem posed in 1904 about the topology of objects called manifolds. A manifold of dimension \(n\) is a geometric object equipped with a topological structure such that every point has a neighborhood that looks like (is homeomorphic to) \(n\)-dimensional Euclidean space, for some \(n.\) The standard example is a sphere, the surface of a ball embedded in three-dimensional space. An ant on the surface of a sphere thinks that he is standing on flat ground, as the curvature of the sphere is not observable locally. So a sphere is a \(2\)-manifold; the flat ground looks like \(2\)-dimensional Euclidean space. Another example of a \(2\)-manifold is a (one-holed) torus. Two manifolds are considered to be different if one cannot be continuously deformed into the other. One way to see that the torus is different from the \(2\)-sphere is that loops on the sphere can all be contracted on the sphere to a point (imagine a rubber band on the surface of a sphere--it can be pulled to the top of the sphere without breaking the band or leaving the sphere), but loops on a torus cannot (e.g. the loop on the top of the torus, or one of the black loops in the picture). A fundamental question in the theory of manifolds is the classification problem: is there a way to characterize when two manifolds are the same, without having to explicitly write down the map that identifies them? That is, is there a set of properties such that any two manifolds that share all these properties must be the same? The Poincare conjecture states that any closed (boundaryless) \(n\)-manifold which is homotopy equivalent to the \(n\)-sphere must be the \(n\)-sphere. (Homotopy equivalence is a notion that is strictly weaker than being the same, in general.) This is relatively easy for \(n=1,2.\) It was proved for \(n\ge 5\) by Stephen Smale in the 1960s, and for \(n=4 \) by Michael Freedman in 1982. Both mathematicians were given Fields Medals, the highest honor a mathematician can receive. The case \(n=3\) is equivalent to the following statement: Any simply connected, closed 3-manifold is the same as the 3-sphere. Here simply connected means intuitively that the manifold has no holes; a loop on its surface can always be contracted to a point. As \(n=3\) was the only case left to be proved, this was the statement of the Poincare conjecture when it was posed as a Millennium Problem. The conjecture was proved in 2003 by the Russian mathematician Grigori Perelman, using ideas of Richard Hamilton from the early 1980s. Hamilton suggested using a vector field flow called the Ricci flow to solve the problem, and demonstrated its efficacy by proving special cases of Poincare's conjecture. Perelman announced his solution of the problem in a series of papers in 2002 and 2003. Peer review confirmed that his proof was correct, and in 2006 he was offered the Fields Medal for his work. Perelman turned down the Fields Medal and also refused to accept the Clay Millennium Prize when it was officially offered to him in 2010, saying that his contributions were no more significant than Hamilton's. His work is by all accounts quite original and accomplished; despite his apparent modesty and shunning of the spotlight, his proof of the Poincare conjecture will be famous for a very long time.